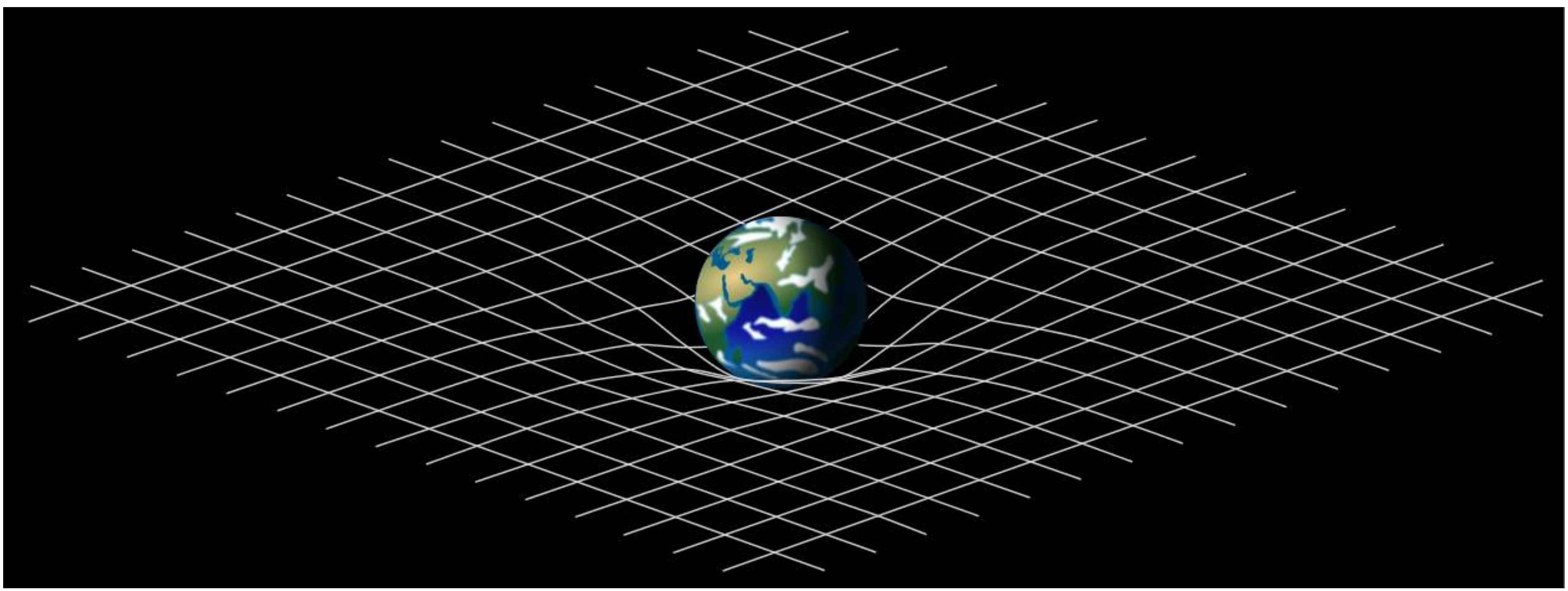
Spacetime coordinates vs. physical quantities

- *Coordinates* are labels used to identify points in 4D spacetime, e.g. (t, x, y, z)
- Coordinates can be chosen arbitrarily (e.g., Cartesian, spherical, cylindrical, … for spatial part)
- Need a "tool" / "function" to convert coordinates into *physical quantities* that can be measured by clocks and meter sticks, *proper distance* and *proper time*
- This function is called the *metric* and defines the *curvature* of spacetime



Spacetime metric

- Metric is a 4×4 matrix $g_{\mu\nu}$ defined such that spacetime *line element*

$$(ds)^2 = \sum_{\mu,\nu} g_{\mu\nu} dx_\mu dx_\nu$$

- E.g., *flat spacetime* — *Minkowski metric* of special relativity:

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$
$$g_{00} = 1, \;\; g_{11} = -1, \;\; g_{22} = -1, \;\; g_{33} = -1$$

if using Cartesian coords for spatial

$$(ds)^2 = (cdt)^2 - (dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2$$
$$g_{00} = 1, \;\; g_{11} = -1, \;\; g_{22} = -r^2, \;\; g_{33} = -r^2 \sin^2\theta$$

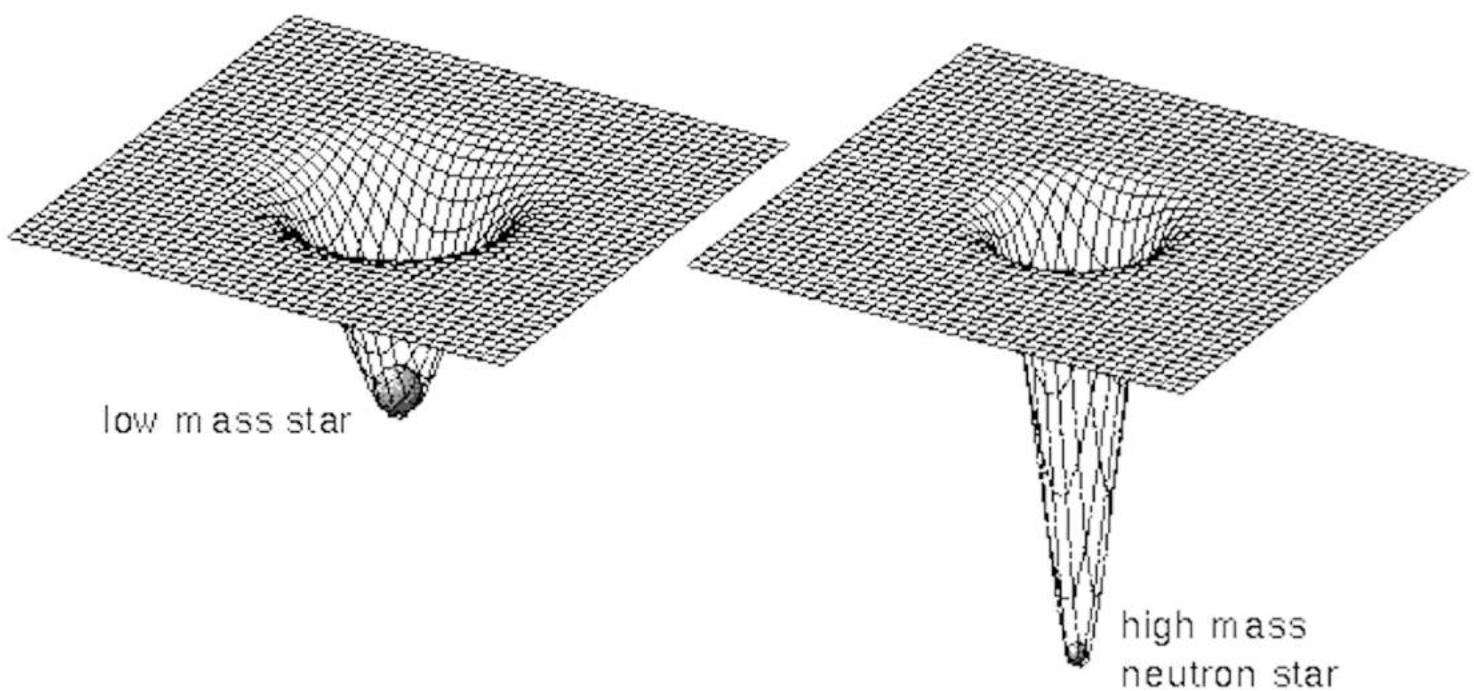
if using spherical coords for spatial

Energy & momentum determine the metric via Einstein's field equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

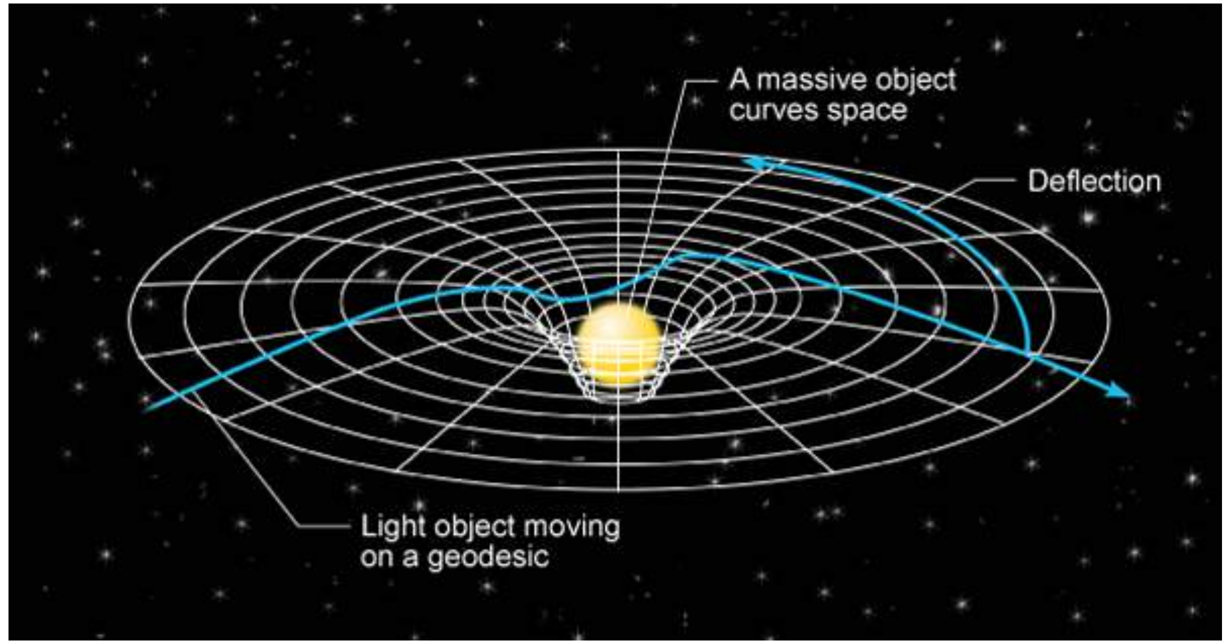
<u>Einstein tensor</u>
function of metric and its 1st and 2nd derivatives

<u>Stress-energy tensor</u>
describes how energy and momentum are distributed in space



Motion of test particles follows geodesics

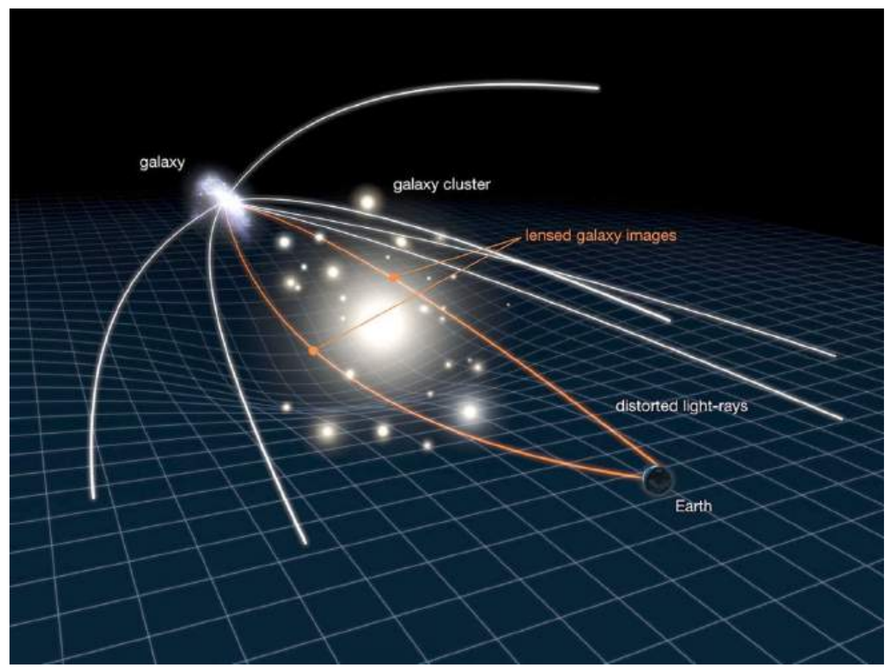
- Locally straightest paths



- In curved spacetimes,

  correspond to e.g.

  - planetary orbits
  - light deflected by foreground stars or galaxies



Schwarzschild metric

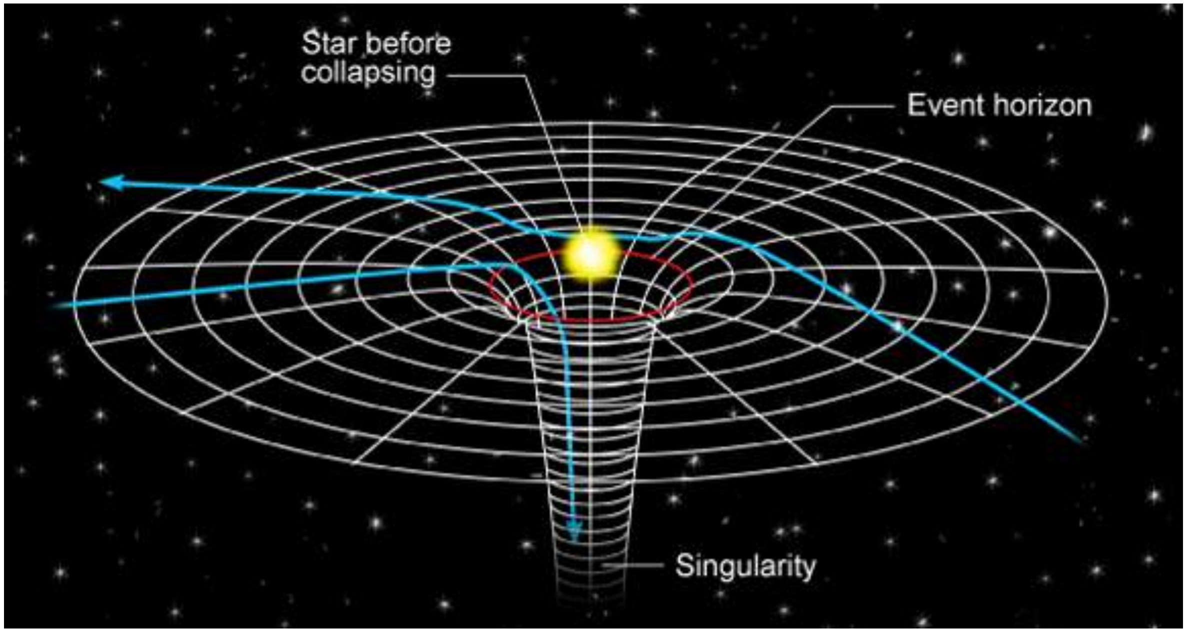
- Vacuum solution around spherically symmetric massive objects (Sun, NS, BH, …):

$$(ds)^2 = \left(1 - \frac{2GM}{rc^2}\right)(cdt)^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1}(dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2$$

- Matter or light that enters **event horizon** or **Schwarzschild radius**

$$r_s = \frac{2GM}{c^2} = 3\,\mathrm{km}\frac{M}{M_\odot}$$

of black holes cannot escape



Black hole accretion power

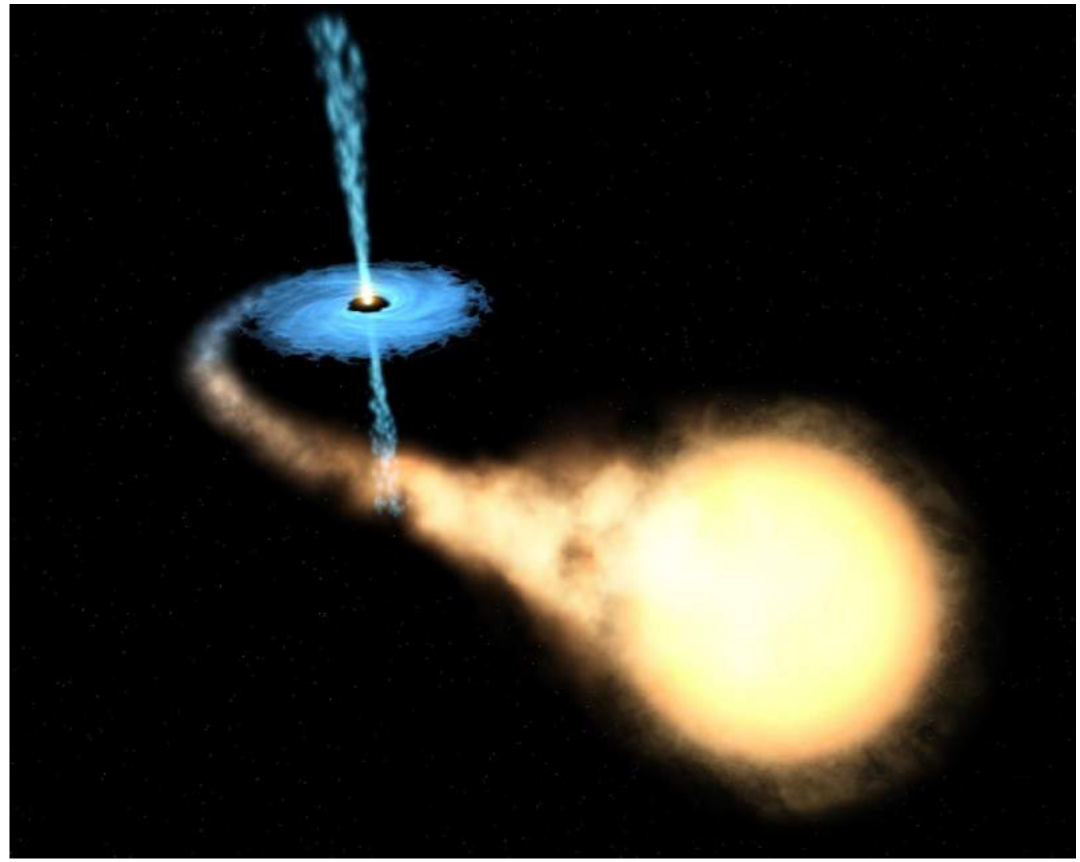
- ~10^8 $M_{\rm BH}$~5-30 $M_{\rm sun}$ BHs in Milky Way (out of ~10^{11} stars)
- In ***X-ray binaries***, infalling gas circularizes and forms ***accretion disk***
- Disk viscosity transports angular momentum, heating the disk and bringing the gas into the BH
- Radiative efficiency

  $$L_{\rm acc} \sim 0.1\dot{M}c^2$$

  exceeds H-burning efficiency

  $$L_{\rm nuc} \sim 0.007\dot{M}c^2$$

  by factor >10!

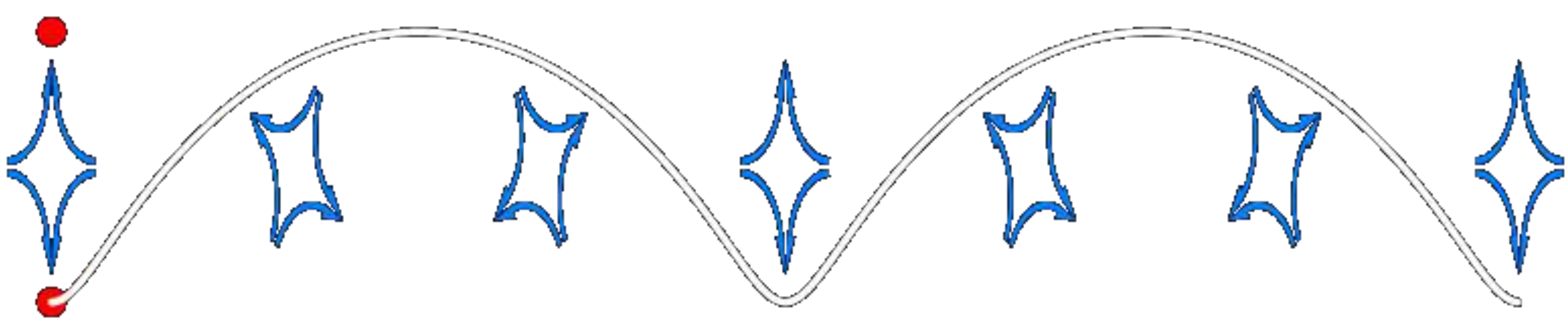


Gravitational waves

- Analogous to E&M radiation but ***generated by accelerated masses*** instead of accelerated electric charges
- ***Oscillations in the spacetime metric***, i.e. changes in physical distances
- ***Frequency*** is characteristic frequency of the source, e.g. ~1/orbital period
- Main stellar sources:

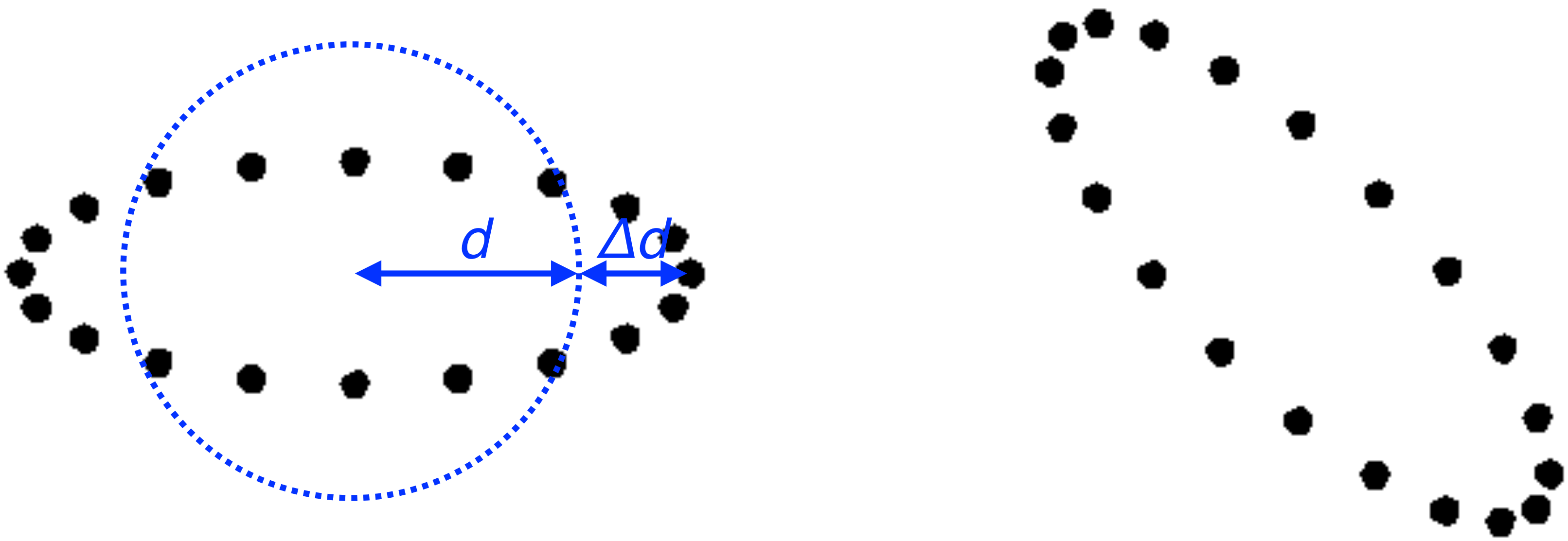
  - NS-NS, NS-BH and BH-BH binaries

  - asymmetric core collapse SNe



Polarizations & strain

- General GW is superposition of "+" and "x" ***polarizations***. Effects on rings of test particles:

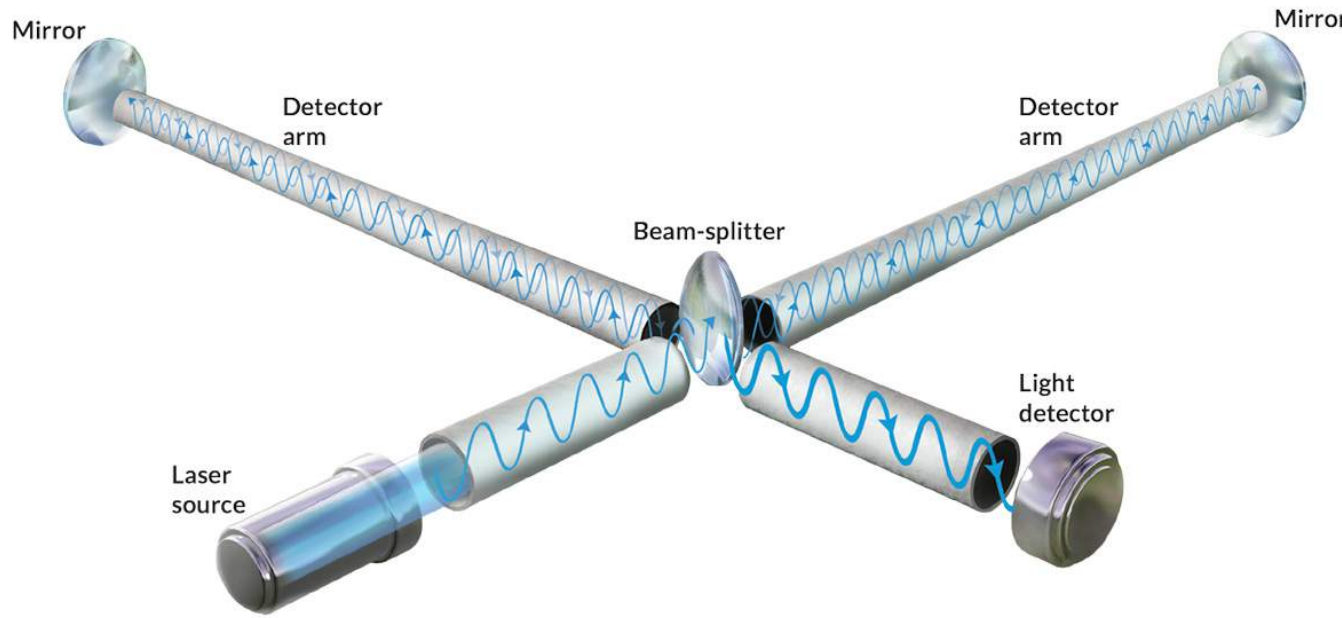


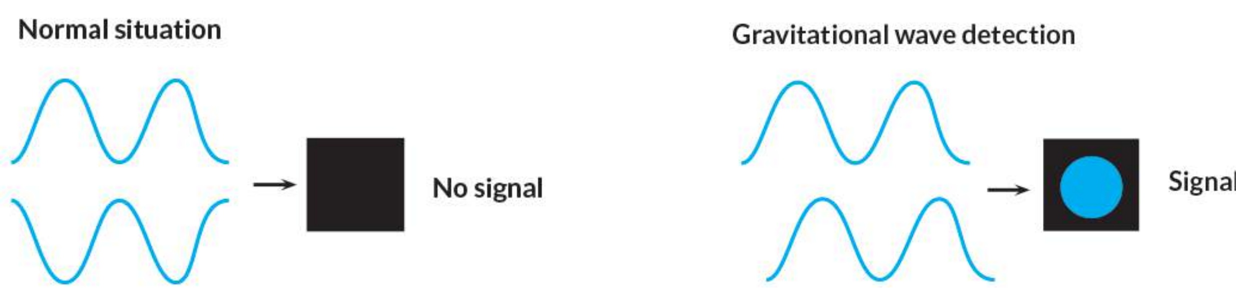
- ***Strain*** *h* = fractional change *2Δd/d*. Order-of-magnitude estimate:

$$h \sim \left(\frac{\text{system's Schwarzschild radius}}{\text{distance}}\right) \times \left(\frac{\text{characteristic velocity}}{c}\right)^2$$

Gravitational wave detectors

- Astrophysical strains are tiny, e.g. h=10^{-21} for first aLIGO discovery of merging binary BHs
- LIGO observatory is based on a laser Michelson interferometer





September 14, 2015: Advanced LIGO discovery of merging binary black holes